Kelvin-Helmholtz and Rayleigh-Taylor Instability of Two Superimposed Magnetized Fluids with Suspended Dust Particles

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The effect of a magnetic field and suspended dust particles on both the Kelvin-Helmholtz (K-H) and the Rayleigh-Taylor (R-T) instability of two superimposed streaming magnetized plasmas is investigated. The magnetized fluids are assumed to be incompressible and flowing on top of each other. The usual magnetohydrodynamic (MHD) equations are considered with suspended dust particles. The basic equations of the problem are linearized and the dispersion relation is obtained using normal mode analysis by applying the appropriate boundary conditions. The general dispersion relation is found to be modified due to the presence of the suspended dust particles and of the magnetic field. The effect of the magnetic field appears in the dispersion relation if three-dimensional perturbations of the system are considered. The general conditions of the K-H instability as well as the R-T instability are derived for the considered medium. The stability of the system for both cases is discussed by applying the Routh-Hurwitz criterion. Numerical analysis is performed to show the effect of various parameters on the growth rates of the K-H and R-T instabilities. Three different cases of the present configurations are considered and the conditions of instability are obtained. It is found that the conditions for the K-H and R-T instabilities depend on the magnetic field, on the suspended dust particles and on the relaxation frequency of the particles. The magnetic field and particle density have stabilizing influence, while the density difference between the fluids has a destabilizing influence on the growth rate of the K-H and R-T configurations.

Key words: Magnetohydrodynamics (MHD); Plasma Instability; Rayleigh-Taylor Instability; Kelvin-Helmholtz Instability; Suspended Dust Particles.

1. Introduction

The Kelvin-Helmholtz (K-H) instability is the instability of the plane interface between two superimposed fluids flowing on top of each other with a relative horizontal velocity. It is widely discussed to explain many phenomena viz. magnetic reconnection processes in solar and magnetospheric dynamics, in astrophysical jet simulation, high-β plasma processes, magnetic confinement, auroras and magnetopause stability, and clusters of galaxies in astrophysical plasma [1 – 3]. The effects of the magnetic field, the surface tension, rotation, variable viscosity, and many other parameters have been mostly discussed in the context of the magnetohydrodynamic (MHD) K-H instability for incompressible fluid plasmas [4 – 8]. Chengsen et al. [9] have studied the combination of the Rayleigh-Taylor (R-T) instability (which arises when the heavy fluid is supported by the light fluid) and the K-H instability of compressible fluids. The K-H instability in a rotating ideally conducting inhomogeneous plasma has been investigated by Kumar et al. [10]. Chhajlani and Purohit [11] have studied the K-H instability of superimposed hydromagnetic fluids of different densities with finite resistivity. Uberoi [12] has investigated the finite thickness and the angle effect on the marginal K-H instability considering the three-layered structure of the plasma region viz. the magneto-sheath, the boundary layer and the magnetosphere. Chhajlani and Vyas [13] have studied the K-H instability problem in an oblique magnetic field using MHD equations. D'Silva and Choudhuri [14] have discussed the effect of the K-H instability on rising flux tubes in the convection zone. The K-H instability for the magnetosphere boundary layer region is studied by Parhi [15] and Miura [16]. Recently, Watson et al. [17] have investigated the K-H instability due to shear flow in a weakly ionized medium. Thus we find that the K-H instability is currently discussed in different configurations and in different kinds of fluids with velocity shear or with two different velocities at the interface.

Recent spacecraft observations have confirmed that dust particles play an important role in the dynamics of the Martian atmosphere as well as variations of the temperature of the weather. The presence of suspended dust particles in a gas is more realistic in astrophysical situations. In geophysical situations, the fluid is not pure but may be permeated with suspended particles. Michael [18] has investigated the K-H instability of a dusty gas using Stokes' drag force formula. In this direction, the effect of suspended dust particles is widely investigated in fluid dynamics for the discussion of flow and stability problems. Hans [19] has reviewed the K-H instability in a composite medium with neutral particles and finite Larmor radius (FLR) corrections. Kumar [20] has made a study of the implication of suspended particles on the K-H instability in permeable media and found that the critical wavelength decreases due to the presence of the suspended particles. More recently El-Sayed [21] has investigated the K-H instability for two viscoelastic superimposed conducting fluids permeated with suspended particles in a porous medium in the presence of magnetic field. El-Sayed [22] has also studied the combined effects of viscosity, FLR corrections and suspended particles on the K-H instability of two superimposed incompressible fluids. The effect of suspended dust particles, the magnetic field, and the rotation on the R-T instability of a Rivilin-Ericksen elastico-viscous fluid is also discussed [23, 24]. Sharma and Chhajlani [25, 26] have made an investigation of the R-T instability of two superimposed magnetized conducting plasmas with rotation, FLR corrections and neutral particles. Along with this El-Sayed [27] has discussed the K-H instability of two superimposed viscous and streaming dielectric fluid permeated with suspended particles through a porous medium under the influence of a tangential electric field.

Along with this, Sanghvi and Chhajlani [28] have investigated the R-T configuration of a stratified plasma in the presence of suspended particles. Sharma and Sunil [29] have studied the thermal instability of a viscoelastic fluid with suspended particles in hydromagnetics. Sunil and Chand [30] have investigated the R-T instability of plasma in the presence of a variable magnetic field and suspended particles in porous media. Sharma and Kumari [31] have studied the stability of a stratified fluid in a porous medium in the presence of suspended particles and a variable horizontal magnetic field.

In this direction, Sanghvi and Chhajlani [32] have studied the hydromagnetic K-H instability of two superimposed streaming fluids acted upon by a uniform magnetic field transverse to the flow direction of streaming in the presence of suspended particles and FLR corrections. In their work we find that perturbations are considered only in y-direction and the magnetic field is assumed to be in x-direction. Due to the limitations in considering perturbations they could not achieve the effect of a magnetic field in the dispersion relation. Thus for the complete understanding of the R-T and K-H instabilities of two superimposed magnetized fluids in the presence of various parameters, we must consider perturbations in both the direction. In the light of above studies, the object of the work presented in this paper is to investigate the influence of suspended dust particles on the joint K-H and R-T configurations of two superimposed streaming fluids for general perturbation in the presence of a uniform magnetic field.

2. Basic Equations of the Problem

Let us consider two semi-infinite homogeneous fluids separated by a plane interface (of negligible thickness) at $z = 0$. The fluids in the regions $z < 0$ and $z > 0$ are, respectively, denoted by the subscripts 1 and 2 (see Fig. 1). Each region is permeated with non-conducting

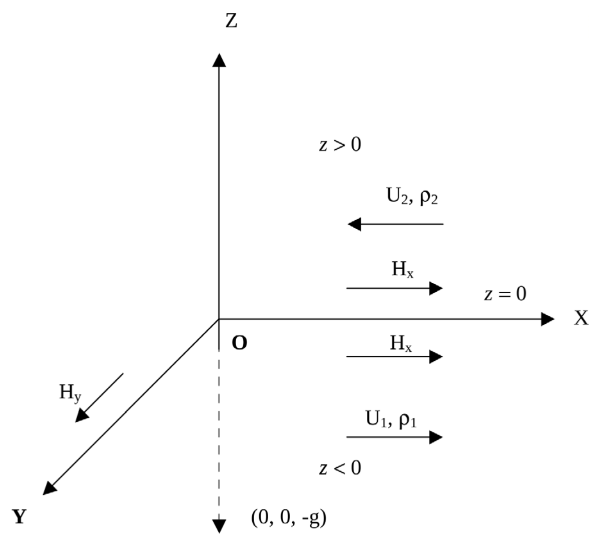

Fig. 1. Schematic diagram of the configuration.

suspended dust particles of uniform size, spherical in shape and with the same number density. Thus the medium can be assumed as a uniform mixture of an infinitely conducting, incompressible fluid and suspended dust particles. The mixture of the hydromagnetic fluid and the suspended dust particles is streaming together in the presence of a uniform external magnetic field $\boldsymbol{H}(H_x,H_y,0)$ with flow velocity $\boldsymbol{U}(U,0,0)$ and a downward gravitational field $\boldsymbol{g}(0,0,-g)$.

Let $\boldsymbol{V}$ and N denote the velocity and the number density of the dust particles. It is supposed that the bulk concentration of the dust particles is very small, so that the net effect of the dust particles on the fluid is equivalent to an extra body force $KN(\boldsymbol{V}-\boldsymbol{U})$, where K is a constant given by $K=6\pi a\mu$ (Stokes' drag formula), a being the particle radius and μ is the viscosity of the clean fluid.

On the basis of these assumptions the relevant basic equations of the problem are

$$\rho\left[\frac{\partial \boldsymbol{U}}{\partial t}+(\boldsymbol{U}\cdot\nabla)\boldsymbol{U}\right]=-\nabla p+\frac{1}{4\pi}(\nabla\times\boldsymbol{H})\times\boldsymbol{H}+KN(\boldsymbol{V}-\boldsymbol{U})+\rho\boldsymbol{g}+\mu\nabla^2\boldsymbol{U}, \quad (1)$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{U})=0, \quad (2)$$

$$\nabla\cdot\boldsymbol{U}=0, \quad (3)$$

$$\frac{\partial\boldsymbol{H}}{\partial t}=(\boldsymbol{H}\cdot\nabla)\boldsymbol{U}-(\boldsymbol{U}\cdot\nabla)\boldsymbol{H}, \quad (4)$$

$$\nabla\cdot\boldsymbol{H}=0, \quad (5)$$

where ρ and p denote the density and pressure of the fluid.

In the dynamics of fluid motion, the force exerted by the fluid on the dust particles is equal and opposite to the force exerted by the dust particles on the fluid. The buoyancy force on the particles is neglected, as its stabilizing effect is extremely small. The inter-particle distance is assumed to be very large as compared to the diameter of the particles and so the inter-particle reactions can be ignored. The effects of electrical and magnetic forces on the suspended dust particles are also ignored. Under these restrictions the equations of continuity and motion for such particles are

$$\frac{\partial N}{\partial t}+\nabla\cdot(N\boldsymbol{V})=0 \quad (6)$$

and

$$mN\left[\frac{\partial\boldsymbol{V}}{\partial t}+(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\right]=KN(\boldsymbol{U}-\boldsymbol{V}), \quad (7)$$

where mN is the mass of the dust particles per unit volume.

To investigate the stability of the configuration, we assume the following perturbations in various physical quantities:

$$\begin{aligned}&\boldsymbol{V}=\boldsymbol{U_0}+\boldsymbol{v},\quad \boldsymbol{U}=\boldsymbol{U_0}+\boldsymbol{u},\\ &N=N_0+N\ \ (\text{with } N_0=\text{constant}),\\ &p=p_0+\delta p,\quad \boldsymbol{H}=\boldsymbol{H_0}+\boldsymbol{h},\ \text{ and }\ \rho=\rho_0+\delta\rho,\end{aligned} \quad (8)$$

where the quantities with the subscripts 0 denote equilibrium values and the quantities $\boldsymbol{v}$, $\boldsymbol{u}(u,v,w)$, N, δp, $\boldsymbol{h}(h_x,h_y,h_z)$, and $\delta\rho$ denote the perturbations in the velocity of suspended dust particles, in the flow velocity of the fluid, the number density of the dust particles, the fluid pressure, the magnetic field, and in the density of the fluid, respectively. $\boldsymbol{U_0}(U,0,0)$ is the unperturbed initial velocity of flow of fluid in x-direction. In solving we remove the subscript 0 from all the equilibrium quantities for simplicity.

The linearized perturbation equations of such a medium in the presence of suspended dust particles are

$$\rho\left[\frac{\partial\boldsymbol{u}}{\partial t}+(\boldsymbol{U}\cdot\nabla)\boldsymbol{u}\right]=-\nabla\delta p+\frac{1}{4\pi}(\nabla\times\boldsymbol{h})\times\boldsymbol{H}+KN(\boldsymbol{v}-\boldsymbol{u})+\boldsymbol{g}\delta\rho, \quad (9)$$

$$\frac{\partial}{\partial t}\delta\rho+(\boldsymbol{U}\cdot\nabla)\delta\rho+(\boldsymbol{u}\cdot\nabla)\rho=0, \quad (10)$$

$$\left[\tau\left(\frac{\partial}{\partial t}+\boldsymbol{U}\cdot\nabla\right)+1\right]\boldsymbol{v}=\boldsymbol{u}, \quad (11)$$

$$\frac{\partial\boldsymbol{h}}{\partial t}+(\boldsymbol{U}\cdot\nabla)\boldsymbol{h}=(\boldsymbol{H}\cdot\nabla)\boldsymbol{u}, \quad (12)$$

$$\nabla\cdot\boldsymbol{u}=0\ \text{ and }\ \nabla\cdot\boldsymbol{h}=0, \quad (13)$$

where $\tau=m/K$ denotes the relaxation time for the suspended dust particles. It is also noticed that, in writing (9), we have assumed that the viscous term $\mu\nabla^2\boldsymbol{u}$ is negligible as compared to the viscous drag force $KN(\boldsymbol{v}-\boldsymbol{u})$.

Analyzing the perturbations via expansion into normal mode, we seek solutions of (9)–(13), whose dependence on x, y and t is given by

$$\exp(\mathrm{i}k_x x+\mathrm{i}k_y y+nt), \quad (14)$$

where k_x and k_y are the horizontal wavenumbers ($k^2=k_x^2+k_y^2$) and n is the temporal growth rate of the perturbation.

3. Dispersion Relation

Now eliminating $\boldsymbol{v}$ from (9) with the help of (11) and then employing (14) on (9) to (13), we obtain the following set of equations:

$$\rho\left[n+\mathrm{i}k_xU+\frac{\alpha_0(n+\mathrm{i}k_xU)}{\tau(n+\mathrm{i}k_xU)+1}\right]u = -\mathrm{i}k_x\delta p+\frac{H_y}{4\pi}(\mathrm{i}k_yh_x-\mathrm{i}k_xh_y), \tag{15}$$

$$\rho\left[n+\mathrm{i}k_xU+\frac{\alpha_0(n+\mathrm{i}k_xU)}{\tau(n+\mathrm{i}k_xU)+1}\right]v = -\mathrm{i}k_y\delta p+\frac{H_x}{4\pi}(\mathrm{i}k_xh_y-\mathrm{i}k_yh_x), \tag{16}$$

$$\rho\left[n+\mathrm{i}k_xU+\frac{\alpha_0(n+\mathrm{i}k_xU)}{\tau(n+\mathrm{i}k_xU)+1}\right]w = -D\delta p-g\delta\rho+\frac{H_x}{4\pi}(\mathrm{i}k_xh_z-Dh_x) + \frac{H_y}{4\pi}(\mathrm{i}k_yh_z-Dh_y), \tag{17}$$

$$(n+\mathrm{i}k_xU)\delta\rho=-w(D\rho), \tag{18}$$

$$(n+\mathrm{i}k_xU)\boldsymbol{h}=(\mathrm{i}k_xH_x+\mathrm{i}k_yH_y)\boldsymbol{u}, \tag{19}$$

$$\mathrm{i}k_xu+\mathrm{i}k_yv+Dw=0, \tag{20}$$

$$\mathrm{i}k_xh_x+\mathrm{i}k_yh_y+Dh_z=0, \tag{21}$$

where $\alpha_0=mN/\rho$ denotes the mass concentration of the particles and $D=\mathrm{d}/\mathrm{d}z$.

Multiplying (15) and (16) by $-\mathrm{i}k_x$ and $-\mathrm{i}k_y$, respectively, and adding the results using (20), we get

$$\left[n+\mathrm{i}k_xU+\frac{\alpha_0(n+\mathrm{i}k_xU)}{\tau(n+\mathrm{i}k_xU)+1}\right]\rho(Dw) = -k^2\delta p+\frac{1}{4\pi}\left[H_y(k_xk_yh_x-k_x^2h_y) + H_x(k_xk_yh_y-k_y^2h_x)\right]. \tag{22}$$

Now eliminating δp between (17) and (22) and using (18)–(21), we get

$$[D(\rho Dw)-k^2\rho w]\left[n+\mathrm{i}k_xU+\frac{\alpha_0(n+\mathrm{i}k_xU)}{\tau(n+\mathrm{i}k_xU)+1}\right] + \frac{(H_xk_x+H_yk_y)^2}{4\pi(n+\mathrm{i}k_xU)}(D^2-k^2)w+\frac{gk^2(D\rho)}{n+\mathrm{i}k_xU}w=0. \tag{23}$$

Equation (23) is a general dispersion relation incorporating the effects of the magnetic field and of the suspended dust particles. It should be remarked here that the density of the suspended dust particles in the both regions is assumed to be the same.

Consider the case of two superimposed fluids of densities ρ_1 (lower fluid) and ρ_2 (upper fluid), separated by a horizontal boundary at $z=0$. Let the streaming velocities of the two fluids be $\boldsymbol{U_1}(U_1,0,0)$ and $\boldsymbol{U_2}(U_2,0,0)$ then, in each of the two regions of constant densities, (23) becomes

$$(D^2-k^2)w=0. \tag{24}$$

Since w must be bounded both when $z\to-\infty$ (in the lower fluid) and $z\to+\infty$ (in the upper fluid), the appropriate solutions of (24) can be written as

$$w_1=A(n+\mathrm{i}k_xU_1)\exp(kz) \quad (z<0), \tag{25}$$

$$w_2=A(n+\mathrm{i}k_xU_2)\exp(-kz) \quad (z>0), \tag{26}$$

where A is the constant.

Following Chandrasekhar [4], the boundary conditions across the interface of the two fluids are:

(i) The normal component of the velocity is continuous, thus we get

$$\frac{w_1}{(n+\mathrm{i}k_xU_1)}=\frac{w_2}{(n+\mathrm{i}k_xU_2)}. \tag{27}$$

(ii) The total pressure should be continuous. This condition can be obtained by integrating (23) across the interface $z=0$.

(iii) The normal component of the magnetic field is continuous. This reduces to condition (i).

To satisfy the boundary condition (ii), integrating (23) across the interface $z=0$, we obtain

$$\Delta_0\left\{\rho Dw\left[n+\mathrm{i}k_xU+\frac{\alpha_0(n+\mathrm{i}k_xU)}{\tau(n+\mathrm{i}k_xU)+1}\right]\right\} + \frac{H_x^2k_x^2}{4\pi}\Delta_0\left(\frac{Dw}{n+\mathrm{i}k_xU}\right)+\frac{H_y^2k_y^2}{4\pi}\Delta_0\left(\frac{Dw}{n+\mathrm{i}k_xU}\right) + \frac{k_xk_yH_xH_y}{2\pi}\Delta_0\left(\frac{Dw}{n+\mathrm{i}k_xU}\right) + gk^2\Delta_0(\rho)\left(\frac{w}{n+\mathrm{i}k_xU}\right)_{z=0}=0, \tag{28}$$

where $\Delta_0(f)$ is the jump that a quantity f experiences at the interface $z=0$ and $[w/(n+\mathrm{i}k_xU)]_{z=0}$ is the unique value that this quantity has at $z=0$.

Using the values of w_1 and w_2 from (25) and (26) in (28), we obtain the dispersion relation

$$n^2+2\mathrm{i}nk_x(\beta_1U_1+\beta_2U_2)-k_x^2(\beta_1U_1^2+\beta_2U_2^2) -gk(\beta_2-\beta_1)+2(k_xV_A+k_yV_B)^2+\frac{\alpha_1\beta_1(n+\mathrm{i}k_xU_1)^2}{[\tau(n+\mathrm{i}k_xU_1)+1]} +\frac{\alpha_2\beta_2(n+\mathrm{i}k_xU_2)^2}{[\tau(n+\mathrm{i}k_xU_2)+1]}=0, \quad (29)$$

where $\alpha_1=mN/\rho_1$, $\alpha_2=mN/\rho_2$, $\beta_1=\rho_1/(\rho_1+\rho_2)$, $\beta_2=\rho_2/(\rho_1+\rho_2)$, and

$$V_{A,B}^2=H_{x,y}^2/4\pi(\rho_1+\rho_2). \quad (30)$$

Equation (29) represents the influence of the suspended dust particles and of the magnetic field on the combined hydromagnetic K-H and R-T instability of two superimposed fluids. The effect of the suspended dust particles enters into the dispersion relation (29) through two parameters, α_0 and τ, measuring the mass concentration and the relaxation time of the particles. If we ignore the effects of the transverse magnetic field ($V_B=0$) and the perturbation in x-direction ($k_x=0$) in the dispersion relation (29), this reduces to Hans' [19] result excluding FLR corrections, showing the same behavior of neutral particles taken by him and suspended dust particles taken by us in the present problem. In the absence of a transverse magnetic field ($V_B=0$) and considering perturbation only in y-direction ($k_x=0$), this dispersion relation reduces to Sanghvi's and Chhajlani's [32] formula excluding the FLR corrections in that case. They have considered the magnetic field in x-direction and the perturbation in y-direction and they did not get any contribution of the magnetic field in the dispersion relation, whereas in the present case the magnetic field terms are well appearing in the dispersion relation. Thus the results in the present problem are an improvement due to the presence of the magnetic field and because of the three-dimensional perturbations.

4. Discussions

In order to study the effect of the magnetic field on the conditions of the K-H and R-T instabilities and on the growth rates we consider three special cases.

4.1. General Configuration with Magnetic Field

In this subsection we consider the configuration in the absence of suspended dust particles but with the magnetic field. In this case $(\alpha_1=\alpha_2=0)$ the dispersion relation (29) reduces to

$$n^2+2\mathrm{i}nk_x(\beta_1U_1+\beta_2U_2)-k_x^2(\beta_1U_1^2+\beta_2U_2^2) +gk(\beta_1-\beta_2)+2(k_xV_A+k_yV_B)^2=0. \quad (31)$$

We just get the terms showing the effects of the magnetic field in the dispersion relation. In the case of Sanghvi and Chhajlani [32], considering two-dimensional perturbations, the magnetic field did not appear in the dispersion relation. Thus in the present case the magnetic field effects on the condition of instability.

The roots of (31) are given by

$$n=-\mathrm{i}k_x(\beta_1U_1+\beta_2U_2)\pm\{k_x^2\beta_1\beta_2(U_1-U_2)^2 -[gk(\beta_1-\beta_2)+2(k_xV_A+k_yV_B)^2]\}^{1/2}. \quad (32)$$

If we consider the magnetic field effect in the streaming direction only (i. e. $V_A\neq 0$, $V_B=0$), then relation (32) reduces to Chandrasekhar's [4] formula (cf. Eq. 204, Chapt. X). Thus his findings have been modified by the presence of the magnetic field transverse to the direction of streaming and by suspended dust particles. In the absence of both transverse and longitudinal magnetic fields, (32) reduces to (29) of Sanghvi and Chhajlani [32]. If we ignore the effect of the transverse magnetic field and of gravity (i. e. $V_B=g=0$), relation (32) reduces to the expression given by D'Silva and Choudhuri [14] (cf. Eq. 2).

(a) Stable case $(\beta_1>\beta_2)$

If $\beta_1>\beta_2$, one can observe from (32) that the effect of the magnetic field is to suppress the K-H instability if

$$k_x^2\beta_1\beta_2(U_1-U_2)^2 \leq gk(\beta_1-\beta_2)+2(k_xV_A+k_yV_B)^2. \quad (33)$$

In the absence of a transverse magnetic field ($V_A\neq 0$, $V_B=0$), we obtain the same condition of instability as given by Chandrasekhar [4] (cf. Eq. 205, Chapt. X). Hence the condition of instability is modified due to the presence of a transverse magnetic field. Under the above condition, (32) will not allow any real positive root of n, which implies stability of the system. Thus we conclude that the considered K-H system is stabilized for the wavenumbers determined by the condition (33).

Also, we find instability if

$$k_x^2\beta_1\beta_2(U_1-U_2)^2 > [gk(\beta_1-\beta_2)+2(k_xV_A+k_yV_B)^2]. \quad (34)$$

Hence for a given difference in velocity (U_1-U_2) and a given direction of the wave vector $\boldsymbol{k}$, instability occurs for all wavenumbers

$$k > g(\beta_1-\beta_2)\left[\beta_1\beta_2(U_1-U_2)^2\sin^2\theta - 2(V_A^2\sin^2\theta + V_B^2\cos^2\theta + 2V_AV_B\sin\theta\,\cos\theta)\right]^{-1}, \quad (35)$$

where θ is the angle between the direction of $\boldsymbol{k}$ and H_y.

(b) Unstable case $(\beta_1<\beta_2)$

If $\beta_1<\beta_2$, it is clear from (32) that the effect of both the transverse and the longitudinal magnetic field will suppress the K-H instability if

$$2(k_xV_A+k_yV_B)^2 > k_x^2\beta_1\beta_2(U_1-U_2)^2+gk(\beta_2-\beta_1). \quad (36)$$

The system is therefore stable under the restriction (36), if

$$2(k_xV_A+k_yV_B)^2 < k_x^2\beta_1\beta_2(U_1-U_2)^2 + gk(\beta_2-\beta_1), \quad (37)$$

then the K-H configuration remains unstable since one of the roots of (31) is complex with a positive real part.

Thus for the unstable R-T case $\beta_2>\beta_1$, the configuration is stable or unstable according to $2(k_xV_A+k_yV_B)^2$, being greater than or smaller than $[k_x^2\beta_1\beta_2(U_1-U_2)^2+gk(\beta_2-\beta_1)]$. In the absence of magnetic field, (31) has at least one complex root with positive real part and so the system is unstable for $\beta_2>\beta_1$.

Therefore the magnetic field has a stabilizing effect and completely stabilizes the wavenumber band $k<k^*$, where

$$k^* = g(\beta_2-\beta_1)\big/\left[2(V_A\sin\theta+V_B\cos\theta)^2 - \beta_1\beta_2\sin^2\theta(U_1-U_2)^2\right]. \quad (38)$$

Now considering the case of two streaming fluids in the absence of the gravitational force $(g=0)$, we have

$$n = -\mathrm{i}k_x(\beta_1U_1+\beta_2U_2) \pm \left[k_x^2\beta_1\beta_2(U_1-U_2)^2 - 2(k_xV_A+k_yV_B)^2\right]^{1/2}. \quad (39)$$

Thus, in the absence of gravity the system is stable or unstable according to $2(k_xV_A+k_yV_B)^2$ being greater or smaller than $k_x^2\beta_1\beta_2(U_1-U_2)^2$.

Corresponding to the case of static fluids $(U_1=U_2=0)$ under gravity (R-T configuration) we obtain from (31)

$$n^2+gk\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)+2(k_xV_A+k_yV_B)^2=0. \quad (40)$$

In the absence of a transverse magnetic field $(V_A\neq 0, V_B=0)$, the result reduces to Chandrasekhar's [4] finding (cf. Eq. 234, Chapt. X).

If $(\beta_1>\beta_2)$, the system remains always in the stable state. However when $\beta_1<\beta_2$, we find that the system is unstable for all wavenumbers satisfying the condition

$$gk\left(\frac{\rho_2-\rho_1}{\rho_1+\rho_2}\right) > 2(k_xV_A+k_yV_B)^2. \quad (41)$$

4.2. Static Configuration with Suspended Dust Particles

In this subsection we deal with the case of non-streaming hydromagnetic fluids of different densities including suspended dust particles. For that case the dispersion relation (29) reduces to

$$\tau n^3+n^2(1+\alpha_1\beta_1+\alpha_2\beta_2) + n[2\tau(k_xV_A+k_yV_B)^2-\tau gk(\beta_2-\beta_1)] + 2(k_xV_A+k_yV_B)^2-gk(\beta_2-\beta_1)=0. \quad (42)$$

If we ignore the effect of the magnetic field in the above dispersion relation we get the same results as have been obtained by Sharma and Chhajlani [25] in absence of rotation and FLR corrections in that case. Thus the presence of both the longitudinal and the transverse magnetic fields modify the results.

Introducing the relaxation frequency parameter $f_s(=1/\tau)$ of the suspended dust particles and simplifying the above equation, we get

$$n^3+n^2f_s(1+2\alpha') + n[2(k_xV_A+k_yV_B)^2-gk(\beta_2-\beta_1)] + f_s[2(k_xV_A+k_yV_B)^2-gk(\beta_2-\beta_1)]=0, \quad (43)$$

where $\alpha' = mN/(\rho_1+\rho_2)$.

We now consider the dynamical stability of the system by applying the Routh-Hurwitz criterion on the

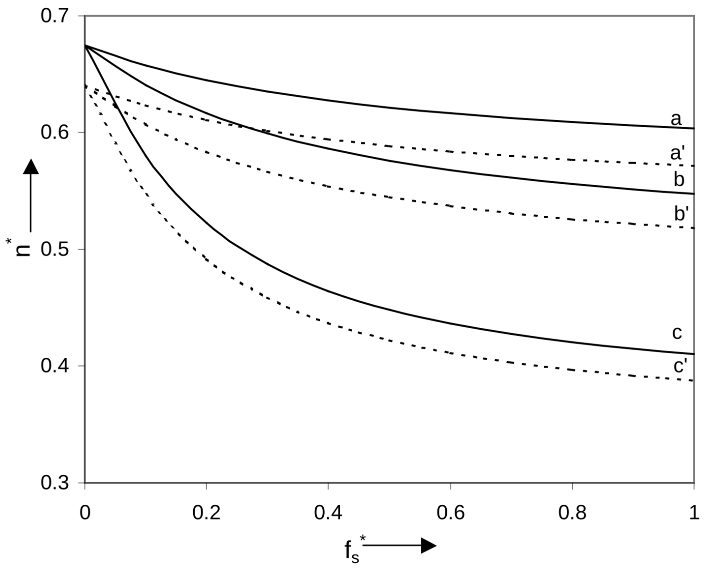

Fig. 2. The growth rate (positive real roots of n^*) of the unstable R-T mode plotted against the relaxation frequency f_s^* with variation in particle density α'.

dispersion relation (43). If $\beta_1 > \beta_2$, then all coefficients of (43) are positive, satisfying the necessary condition of stability. To obtain the sufficient condition we determine the principal diagonal minors of the Hurwitz matrix formed by these coefficients. The principal diagonal minors are

$$\Delta_1 = f_s(1+2\alpha') > 0,$$
$$\Delta_2 = 2\alpha' f_s[2(k_xV_A + k_yV_B)^2 + gk(\beta_1 - \beta_2)] > 0,$$
$$\Delta_3 = 2\alpha' f_s^2[2(k_xV_A + k_yV_B)^2 + gk(\beta_1 - \beta_2)]^2 > 0,$$

which are all positive, thereby satisfying the Routh-Hurwitz criterion. Hence the system represented by (43) is stable if $\beta_1 > \beta_2$.

Let us now consider the alternative case $\beta_1 < \beta_2$. Then the condition of instability, which is independent of the suspended dust particles, is given by

$$2(k_xV_A + k_yV_B)^2 < gk(\beta_2 - \beta_1). \quad (44)$$

Under the above restriction, (43) will necessarily possess one real positive root (n_0), which will give instability to the system.

We obtain the growth rate with increasing relaxation frequency of the particles ($\mathrm{d}n_0/\mathrm{d}f_s$) from (43)

$$\frac{\mathrm{d}n_0}{\mathrm{d}f_s} = -\left[n_0^2(1+2\alpha') + 2(k_xV_A + k_yV_B)^2 - gk(\beta_2 - \beta_1)\right] \Big/ \left[3n_0^2 + 2n_0 f_s(1+2\alpha') + 2(k_xV_A + k_yV_B)^2 - gk(\beta_2 - \beta_1)\right]. \quad (45)$$

The growth rate is found to be negative, if the conditions given below hold simultaneously

$$n_0^2(1+2\alpha') + 2(k_xV_A + k_yV_B)^2 > gk(\beta_2 - \beta_1), \quad (46)$$

and

$$3n_0^2 > gk(\beta_2 - \beta_1). \quad (47)$$

In the present case we are getting the condition of instability influenced by the magnetic field, whereas in Sanghvi and Chhajlani [32] there was no term containing the magnetic field in this condition. From the above conditions we find that the growth rate of the unstable modes decreases with increasing relaxation frequency of the suspended dust particles. Thus we may conclude that, under the restrictions (46) and (47), the suspended dust particles have stabilizing influence on the considered magnetized configuration.

The dispersion relation (43) can be written in dimensionless form using the substitutions

$$n^* = n/\sqrt{gk}, \quad f_s^* = f_s/\sqrt{gk},$$
$$V_A^* = \sqrt{k/g}\, V_A, \text{ and } V_B^* = \sqrt{k/g}\, V_B.$$

Thus we get

$$n^{*3} + n^{*2} f_s^*(1+2\alpha') + n^*[2(V_A^*\sin\theta + V_B^*\cos\theta)^2 - (\beta_2 - \beta_1)] + f_s^*[2(V_A^*\sin\theta + V_B^*\cos\theta)^2 - (\beta_2 - \beta_1)] = 0. \quad (48)$$

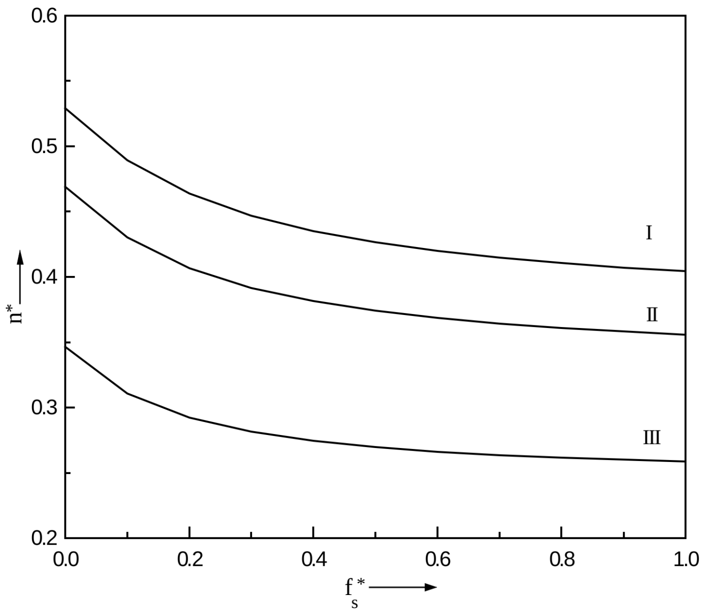

Fig. 3. The growth rate (positive real roots of n^*) of the unstable R-T mode plotted against the relaxation frequency f_s^* with varying magnetic field. Curves I, II and III are plotted for $V_A^* = V_B^* = 0.1$, 0.2 and 0.3, respectively.

In order to study the influence of the suspended particles, the magnetic field and the fluid density on the growth rate of the unstable R-T modes, we have performed numerical calculations of the dispersion relation (48) to locate the roots of n^* (growth rate) against f_s^* (relaxation frequency of the suspended dust particles) for several values of α', V_A^*, V_B^*, θ and difference of densities $(\beta_2 - \beta_1)$.

In Figure 2 couples of solid and dashed lines (a, a'), (b, b') and (c, c') are drawn for $\alpha' = 0.2$, 0.4, and 1.2, respectively, for $(\beta_2 - \beta_1) = 0.5$ and $V_A^* = V_B^* = 0.15$. The solid lines are plotted for an inclination angle $\theta = 0°$, while the dashed lines are represent for $\theta = 45°$. From Figure 2 we see that the growth rate (positive real roots of n^*) decreases with increasing relaxation frequency (f_s^*) of the suspended particles as well as with increasing density (α') of the particles and inclination angle θ, thereby showing a stabilizing influence on the considered R-T configuration. The increase in $f_s (= 6\pi\mu a/m)$ suggests an increase in the size (a) of the particles assuming other parameters to be constant. In other words, as the size of the particles increases, the growth rates of the unstable R-T modes decrease. We also note that the growth rate depends upon the angle between wave vector $\boldsymbol{k}$ and the magnetic field $\boldsymbol{H}(H_x, H_y, 0)$. The growth rate is minimum for larger values of the inclination angle θ.

Figure 3 shows the effect of an uniform magnetic field on the growth rate of the unstable R-T mode. The curves are depicted for various values of V_A^* and V_B^* taking $\alpha' = 0.5$ and $\beta_2 - \beta_1 = 0.3$. We find that as the strength of the magnetic field increases the growth rate of the unstable mode decreases, demonstrating the stabilizing influence of the magnetic field on the growth rate of the system.

In Figure 4 we show the influence of the density difference between the fluids on the growth rate of the unstable R-T mode. The curves represent values of the density difference $\beta_2 - \beta_1$ for fixed parameters $\theta = 90°$, $\alpha' = 0.2$, and $V_A^* = V_B^* = 0.5$. We observe that as we increase the fluids density difference, the growth rate of the unstable mode increases. Hence, an increase of the density of the upper fluid tends to destabilize the system.

From these discussions we find that a system as considered here can be stabilized by increasing the suspended dust particle density and the magnetic field, while it can be made more and more unstable by increasing the density difference between the upper fluid (ρ_2) and the lower fluid (ρ_1).

4.3. K-H Instability with Suspended Dust Particles and Magnetic Field

The dispersion relation (29) is very complex. To discuss the implications of suspended dust particles on the K-H instability, we therefore consider a simple model in which two fluids of the same density are flowing on top of each other with streaming velocities U and $-U$.

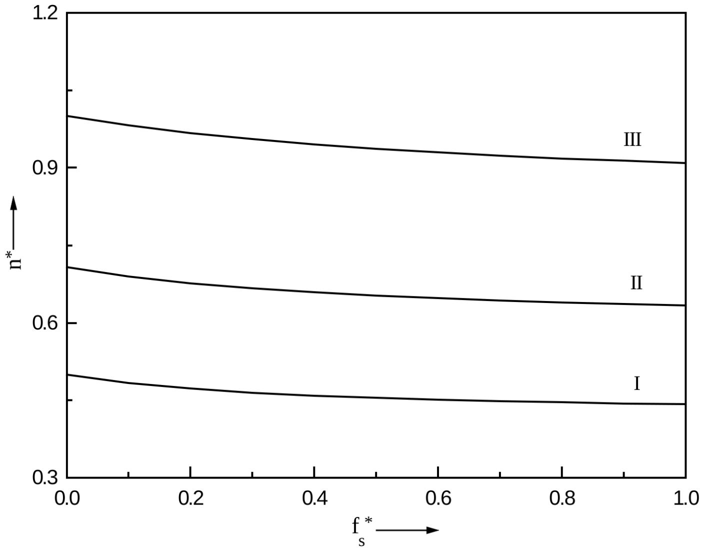

Fig. 4. The growth rate (positive real roots of n^*) of the unstable R-T mode plotted against the relaxation frequency f_s^* with variation in the density difference $\beta_2 - \beta_1$. Curves I, II and III are plotted for $\beta_2 - \beta_1 = 0.1$, 0.2, and 0.3, respectively.

For this case we put $\alpha_1 = \alpha_2 = \alpha_0$, $\beta_1 = \beta_2 = 1/2$, $U_1 = U$ and $U_2 = -U$ in (29).

We obtain the new dispersion relation

$$\begin{aligned}&\tau^2 n^4 + \tau n^3(2+\alpha_0)\\&+n^2[(1+\alpha_0)+2\tau^2(k_xV_A+k_yV_B)^2]\\&+\tau n[k_x^2U^2(\alpha_0-2)+4(k_xV_A+k_yV_B)^2]\\&+[-k_x^2U^2(\tau^2k_x^2U^2+\alpha_0+1)\\&\quad+2(\tau^2k_x^2U^2+1)(k_xV_A+k_yV_B)^2]=0.\end{aligned} \tag{49}$$

In order to discuss the effect of suspended dust particles and a magnetic field on the growth rate of K-H instability against the relaxation frequency of the dust particles, we convert the dispersion relation (49) in an equation for the frequency by introducing $f_s = 1/\tau$ (the relaxation frequency parameter of the suspended dust particles) and we get

$$\begin{aligned}&n^4+n^3f_s(2+\alpha_0)\\&+n^2[f_s^2(1+\alpha_0)+2(k_xV_A+k_yV_B)^2]\\&+nf_s[k_x^2U^2(\alpha_0-2)+4(k_xV_A+k_yV_B)^2]\\&-\{k_x^2U^2[k_x^2U^2+f_s^2(\alpha_0+1)]\\&\quad-2(k_x^2U^2+f_s^2)(k_xV_A+k_yV_B)^2\}=0.\end{aligned} \tag{50}$$

Equation (50) shows the dispersion relation for the K-H instability of two incompressible fluids of the same flow velocity, dust particles density and fluid density including the effects of a magnetic field and of the suspended dust particles. If we neglect the effect of the magnetic field, (50) reduces to the findings of Sanghvi and Chhajlani [32], in absence of FLR corrections in that case. Also, in the absence of a magnetic field and of suspended dust particles the dispersion relation (50) reduces to Chandrasekhar's [4] results. Hence those previous results have been improved to inculde the presence of a magnetic field and of suspended dust particles.

The condition for the K-H instability of the system given by the constant term of (50) is

$$\begin{aligned}&2(k_x^2U^2+f_s^2)(k_xV_A+k_yV_B)^2\\&<k_x^2U^2[k_x^2U^2+f_s^2(\alpha_0+1)].\end{aligned} \tag{51}$$

In writing (50) we have taken notice of the fact that α_0 cannot exceed 1. On examining the dispersion relation (50) we find that in the absence of a magnetic field the system will be unstable, but due to the presence of the magnetic field the condition for the K-H instability is modified by the magnetic field term in the condition for the K-H instability along with the flow velocity. Let n_0 denote the positive root of (51). To study the behavior of the growth rates of unstable modes with respect to f_s, we examine the nature of $\mathrm{d}n_0/\mathrm{d}f_s$ analytically. Then (50) yields

$$\begin{aligned}\frac{\mathrm{d}n_0}{\mathrm{d}f_s}&=-\big[n_0^3(2+\alpha_0)+2n_0^2f_s(1+\alpha_0)\\&\quad+4(k_xV_A+k_yV_B)^2(n_0+f_s)-n_0k_x^2U^2(2-\alpha_0)\\&\quad-2k_x^2U^2f_s(\alpha_0+1)\big]\big/\big\{4n_0^3+3n_0^2f_s(2+\alpha_0)\end{aligned}$$

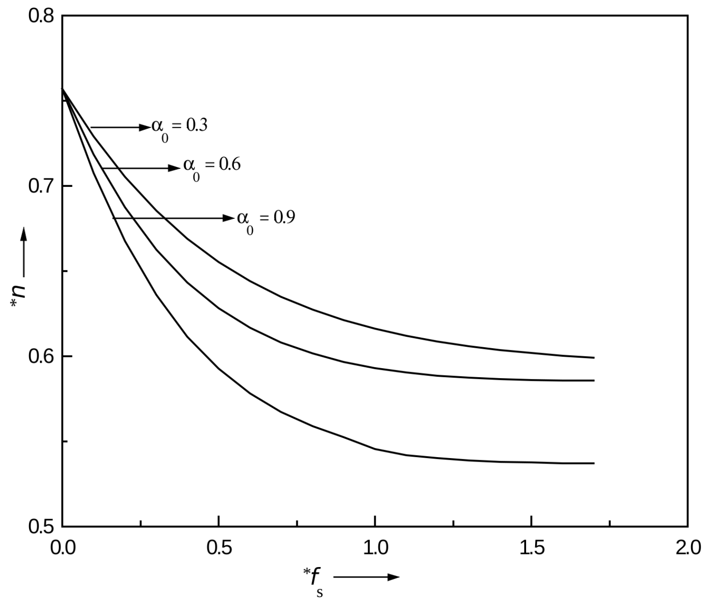

Fig. 5. The growth rates (positive real roots of $^*n$) of the unstable K-H modes plotted against the relaxation frequency $^*f_s$ with varying particle density α_0.

$$+2n_0[f_s^2(\alpha_0+1)+2(k_xV_A+k_yV_B)^2] +4(k_xV_A+k_yV_B)^2-f_sk_x^2U^2(2-\alpha_0)\big\}. \quad (52)$$

Let us now consider the inequalities

$$n_0^3(2+\alpha_0)+2n_0^2f_s(1+\alpha_0) +4(k_xV_A+k_yV_B)^2(n_0+f_s) \gtrless k_x^2U^2[n_0(2-\alpha_0)+2f_s(\alpha_0+1)] \quad (53)$$

and

$$4n_0^3+3n_0^2f_s(2+\alpha_0)+2n_0[f_s^2(\alpha_0+1) +2(k_xV_A+k_yV_B)^2]+4(k_xV_A+k_yV_B)^2 \gtrless f_sk_x^2U^2(2-\alpha_0). \quad (54)$$

If both upper inequalities (53) and (54) are satisfied simultaneously, we find that $\mathrm{d}n_0/\mathrm{d}f_s$ may be negative and if the upper and lower inequalities or vice versa hold, then $\mathrm{d}n_0/\mathrm{d}f_s$ turns out to be positive. From this we conclude that the growth rate can both decrease (for certain wavenumbers) and increase (for different wavenumbers) with the increase of the relaxation frequency parameter of the suspended particles. A similar conclusion regarding the effect of suspended particles has been discussed by Chhajlani et al. [33] in the context of the R-T instability of a stratified plasma in the presence of a uniform horizontal magnetic field.

We introduce some new dimensionless parameters as

$$^*n=n/kU, \quad ^*f_s=f_s/kU,$$
$$^*V_A=V_A/U, \text{ and } ^*V_B=V_B/U.$$

The substitution of these parameters into (50) gives its non-dimensionalized form as

$$^*n^4+{}^*n^3\,{}^*f_s(2+\alpha_0) +{}^*n^2[{}^*f_s{}^2(1+\alpha_0)+2({}^*V_A\sin\theta+{}^*V_B\cos\theta)^2] +{}^*n\,{}^*f_s[(\alpha_0-2)\sin^2\theta+4({}^*V_A\sin\theta+{}^*V_B\cos\theta)^2] +\{2(\sin^2\theta+{}^*f_s{}^2)({}^*V_A\sin\theta+{}^*V_B\cos\theta)^2 -[\sin^2\theta+{}^*f_s{}^2(\alpha_0+1)]\sin^2\theta\}=0. \quad (55)$$

If we put $^*V_A=0$, $^*V_B\neq 0$, and $\theta=90°$ in (55) we recover the dispersion relation already obtained by Sanghvi and Chhajlani [32] for the transverse mode of propagation.

Numerical calculations were performed to locate the roots of $^*n$ from (55) for several values of the parameters $^*f_s$, $^*V_A$, $^*V_B$, α_0 and θ. The results are presented in Figures 5 and 6. This representation gives an idea of the behavior of the effects of the relaxation frequency of the suspended dust particles and the magnetic field on the K-H instability of the considered configuration. The variation of the growth rate with θ has also been included to show the influence of the orientation of the magnetic field with re-

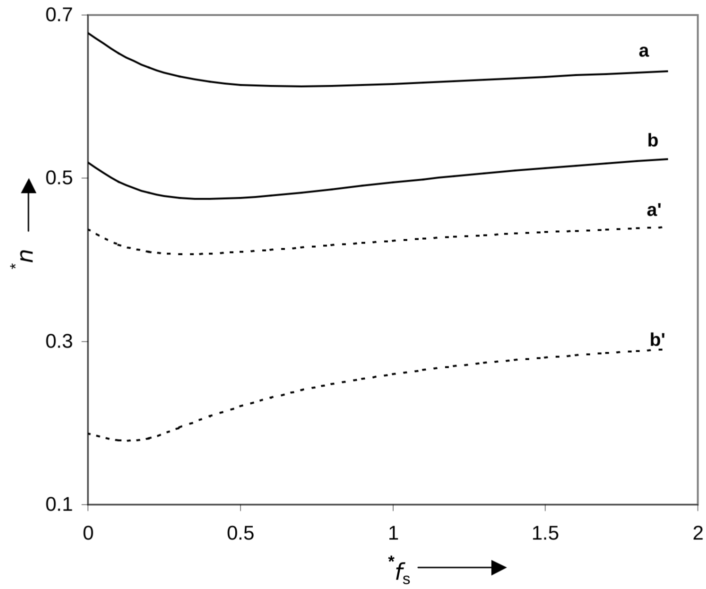

Fig. 6. The growth rates (positive real roots of $^*n$) of the unstable K-H modes plotted against the relaxation frequency $^*f_s$ with varying magnetic field ($^*V_A$ and $^*V_B$). The dashed lines are for $\theta = 30°$ and the solid lines are for $\theta = 45°$, respectively.

spect to the wave vector on the unstable configuration.

In Figure 5 we have plotted the growth rate against the relaxation frequency to study the effect of the particle density on the K-H instability. The curves are plotted for various values of the particle density taking $^*V_A = {}^*V_B = 0.20$ and $\theta = 45°$. It is seen that as one increases the particle density the growth rate of the unstable K-H mode decreases. Thus the particle density has a stabilizing influence on the considered K-H configuration.

In Figure 6 we have plotted the growth rate against the relaxation frequency for magnetic fields $^*V_A = {}^*V_B = 0.1$ and 0.24, respectively. The curves a and a' are for $^*V_A = {}^*V_B = 0.1$, and b and b' are for $^*V_A = {}^*V_B = 0.24$. The particle density α_0 is chosen to be 0.6. We find that as $^*V_A$ and $^*V_B$ increases, the growth rate decreases for all the values of θ and $^*f_s$, showing the stabilizing influence of the magnetic field. The growth rate is also seen to be suppressed for small relaxation frequencies of the suspended particles, although it increases thereafter with increases in $^*f_s$. We conclude that a small relaxation frequency of the suspended particles renders the configuration more stable but as the relaxation frequency increases beyond a critical value, the originally stable configuration becomes unstable. It is also noted from Figure 6, that the growth rate increases as θ increases, for the same $^*V_A$, $^*V_B$, and α_0.

5. Conclusions

In the present paper, a linear analysis of the effect of suspended dust particles on the K-H and the R-T configurations has been carried out in the presence of a uniform magnetic field. The medium is assumed to be incompressible and certain simplifying assumptions are made for the motion of the suspended dust particles. A dispersion relation has been obtained for such a medium, using appropriate boundary conditions, and the effect of the magnetic field appeared in the dispersion relation due to considerating three-dimensional perturbations instead of only two-dimensional ones.

In the case of a stable R-T configuration, the considered system is stabilized for wavenumbers determined by the new condition. Also, the instability occurs for all wavenumbers given by the new condition. But for the case of an unstable R-T configuration the magnetic field has a stabilizing effect on the system. It is found that the growth rate of an unstable R-T mode decreases with increasing relaxation frequency as well as with increasing density of the dust particles, thereby showing a stabilizing influence on the considered R-T configuration. It is also seen that as the density of the upper fluid increases in comparison to the lower fluid (i. e. the density difference between upper and lower fluid increases), the growth rate also increases. Thus the growth rate of the unstable mode will be maximum for two fluids having larger density of the upper fluid.

In case of K-H instability, again the effect of magnetic field is obtained in the K-H instability condition due to our present three-dimensional solution of the problem as compared to previous two-dimensional perturbation, where this effect was not seen in the dispersion relation and in the condition for the K-H instability. We observe that the growth rate of the unstable K-H mode decreases as the dust particle density increases. Also on increasing the magnetic field parameter the growth rate of the system decreases. Hence both the magnetic field and the density of the dust particles have a stabilizing influence on the considered K-H configuration. It is clear that for a given magnetic field the presence of suspended dust particles tends to stabilize the configuration for the relaxation frequencies less than a particular value and for relaxation frequencies greater than this value, the effect is destabilizing.

Thus in the present paper we have studied the effects of magnetic field and of suspended dust particles on the joint K-H and R-T instabilities of two superimposed streaming magnetized incompressible fluids.

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